Effects of Long-Range Coupling on Aggregation

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Numerical simulations of a 2D biharmonic equation $\nabla^4 u = 0$ show that a transition from dense to multibranched growth is *only* a consequence of long-range coupling on the pattern formation of fractal aggregates.

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It has been recently shown that screening, due to free charges, strongly diversifies the pattern that grow in electrostatic fields [1, 2] (see also [3]). By solving the Poisson equation, a transition from dense to multibranched growth has been found which depends on the potentials ϕ^0 and ϕ^i at two boundaries, the distance between them, $L-l$, and an inverse screening length λ. In this note we illustrate an extention of this problem based on the biharmonic (BH) equation $\nabla^4 u = 0$ in 2D isotropic defect-free media. We prove here that the transition from dense to multibranched growth is *only* a consequence of long-range coupling between displacements on pattern formation, and that in the present model the transition appears when the velocity on the growing surface presents a minimum as also occurs in Poisson growth.

These new results are important because of the physical relevance of the BH equation, as follows from the well-known Kuramoto-Sivashinsky (KS) equation that models pattern formation in different physical context, such as chemical reaction-diffusion systems and cellular gas flames in the presence of external stability factors [4, 5]. The BH equation also appears, e.g., when describing the deflection of a thin plate subjected to uniform loading over it surface with fixed edges or within the steady, slow 2D motion of a viscous fluid [6].

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Herein we consider the simplest version of the KS equation, i.e. we assume static solutions. This allows us to include long-range coupling through the discretization of the BH equation on lattice sites involving values of u at *thirteen* mesh points. This is the *crucial* difference with respect to Laplacian and Poisson models in which iterative procedures are carried out around (*four*) next-nearest neighbours (nn) only.

Figure 1 shows the final stage of a BH pattern displaying features, in circular geometry, of a transition from dense to multibranched growth when attaching one particle at each step. Below the transition point $r_l \sim 0.6\,L$, the fractal dimension approaches the value for Laplacian growth within error bars. To generate this BH fractal pattern we have set the derivative boundary condition (DBC), that is necessary along the r-direction, equal to zero and the growth probability P proportional to $\nabla^2 u$ (corresponding to the potential in [1]). Above r_l this figure is a demonstration that long-range coupling is the most relevant aspect for this phenomenon alternative to screening as suggested by Louis et al.

For planar growth we use periodic boundary conditions in the x-direction and estimate the DBC along the y-direction from the analytical solution $3u^0/L$ by fixing $u^i = 0$ and (rescaled) $\phi^0 \equiv \nabla^2 u|_{y=L} \approx 6u^0/L^2$.

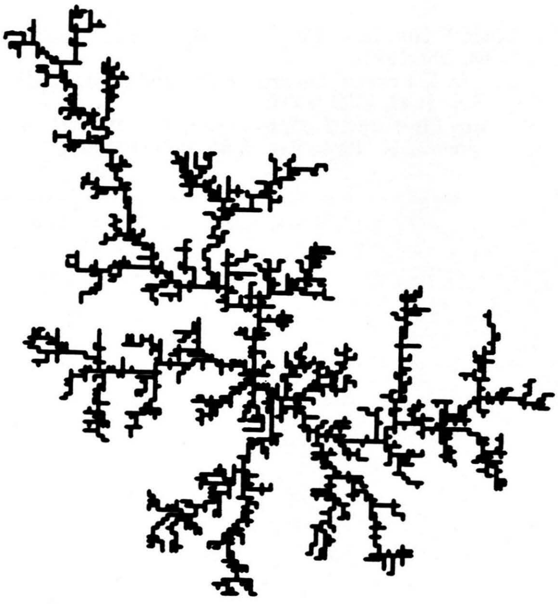

Fig. 1. Biharmonic pattern displaying the transition.

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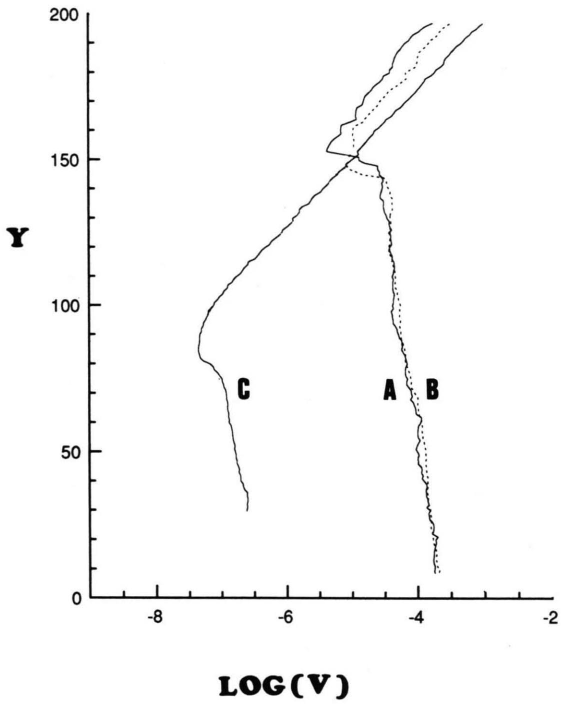

Fig. 2. Growth velocity v along the y-direction proportional to $|\nabla^2 u|$ (A) or displacement $|u|$ (B). (C) is for Poisson growth [1].

In Fig. 2 we plot results for the grow velocity v along the y-direction, which we relate to the averaged value of P, e.g., equal to $\nabla^2 u$ (curve A), to include nn sites, or to the displacement u (curve B). In curve C, v is set proportional to the field $|\phi_{i,j} - \phi^i|$ following [1]. In this geometry we find that the transition appears when v on the growing surfaces presents a minimum, as also occurs in Poisson growth (PG: curve C). However, the BH patterns in the dense region (not shown) are not that denser as in [1, 2]. The reason for this is that for PG the transition occurs at much smaller v than for BH growth, hence an Eden-like pattern can be generated. Above this transition multibranched fractals appear in both models, but for BH growth the transition line depends on the system size. It is remarkable that the three curves in Fig. 2 present parallel slopes above and below their respective transition points.

The transition obtained by numerically solving the BH part of the KS equation might not be necessarily similar to the Hecker transition. But the KS equation can be transformed to look somewhat like the Navier-Stokes equation for a potential flow with negative viscosity which may be somehow related to the recent analysis in [7] concerning electrochemical deposition.

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